Los Angeles Unified School District
Office of Curriculum Instruction and School Support
2015-2016 Common Core Geometry Curriculum Map

Overview of the Common Core Mathematics Curriculum Map

Introduction to the Document:

Welcome to the Los Angeles Unified School District's Common Core Mathematics Curriculum Map. The mathematics curriculum map for Los Angeles Unified School District is developed as a tool for direction and clarification. It is a living document that is interactive and web-based. There are specific, precise links to provide readily accessible resources needed to appropriately meet the rigors of the common core state standards. The curriculum map is intended to be a one-stop tool for teachers, administrators, parents, and other school support personnel. It provides information on the Common Core Standards for Mathematics, assessment sample items, and suggested instructional tools organized into units providing one easy-to-read resource.

Components of the Mathematics Curriculum Map:

The Mathematics Curriculum Map is designed around the standards for mathematics K – 12 which are divided into two sets: Practice Standards and Content standards. The Standards for Mathematical Practice are identical for each grade level. They are the expertise and understanding which the mathematics educators will seek to develop in their students. These practices are also the "processes and proficiencies" to be used as instructional "habits of mind" to be developed at all grade levels. It is critical that mathematical literacy is emphasized throughout the instructional process.

The Mathematics Curriculum Map is grouped into four coherent units by grade level. Each unit clarifies the cluster and specific standards students are to master. In addition, the relevant Mathematical Practices and learning progressions are correlated. These sections of the Mathematics Curriculum Map define the big idea of the unit. These four units are summarized in the **Unit Organizer** which provides the overview for the year.

Instructional components are specified in:

- **Enduring Understandings** are the key understandings/big ideas that the students will learn from the unit of study. These are statements that communicate the learning in a way that engages students.
- **Essential Questions** are based on enduring understandings. They are used to gain student interest in learning and are limited in number. They promote critical or abstract thinking and have the potential of more than one "right" answer. They are connected to targeted standards and are the framework and focus for the unit.
- **Standards**: Targeted (content and skills to be taught and assessed) and supporting (content that is relevant to the unit but may not be assessed; may include connections to other content areas). This includes what students have to know and be able to do (learning targets) in order to meet the standards.

Mathematical literacy is a critical part of the instructional process, which is addressed in:

- **Key Vocabulary** and **Language Goals** which clearly indicate strategies for meeting the needs of EL and SEL students

Planning tools provided are:

- **Instructional Strategies** lead to enduring understandings. They are varied and rigorous instructional strategies to teach content. They are plan experiences that reinforce and enrich the unit while connecting with the standards and assessments. Instructional strategies addresses individual student needs, learner perspectives, integration of technology, learning styles, and multiple intelligences.
- **Resources** and **Performance Tasks** offer concept lessons, tasks, and additional activities for learning.
- **Assessments:** This is also a listing of formative and summative Assessments to guide backwards planning. Student progress in achieving targeted standards/expected learning is evaluated. Entry-level (formative)-based on summative expectations, determine starting points for learning. Benchmark-determine progress of learning, misconceptions, strengths/weaknesses along the learning trajectory.
- **Differentiation** (🕮) falls into three categories:
    - **Universal Design for Learning (UDL) / Universal Access (the approach formerly referred to as Front Loading):** strategies to make the content more accessible to all students, including EL, SEL, Students with Disabilities, and low-achieving students.
    - **Acceleration:** activities to extend the content for all learners, as all learners can have their thinking advanced, and to support the needs of GATE students. These are ideas to deepen the conceptual understanding for advanced learners.
    - **Intervention:** alternative methods of teaching the standards, in which all students can have a second opportunity to connect to the learning, based on their own learning style. They guide teachers to resources appropriate for students needing additional assistance

Using the Mathematics Curriculum Map:

The guide can be thought of as a menu. It cannot be expected that one would do every lesson and activity from the instructional resources provided. To try to teach every lesson or use every activity would be like ordering everything on a menu for a single meal. It is not a logical option. Nor is it possible given the number of instructional days and the quantity of resources. That is why the document is called a "***Mathematics Curriculum Map***" and not a "*Mathematics Pacing Plan*." And, like a menu, teachers select, based on instructional data, which lessons best fit the needs of their students – sometimes students need more time with a concept and at other times, less.

An effective way to use this guide is to review and assess mathematical concepts taught in previous grades to identify potential learning gaps. From there, teachers would map out how much time they feel is needed to teach the concepts within the unit based on the data of their students' needs. For example, some classes may need more time devoted to developing expressions and equations, while another class in the same course may need more focused time on understanding the concept of functions.

The starting point for instructional planning is the standards and how they will be assessed. By first considering how the standards will be assessed, teachers can better select the instructional resources that best build mathematical understanding. There are hundreds of resources available, both publisher- and teacher-created, as well as web-based, that may be used to best teach a concept or skill. Collaborative planning, both within and among courses, is strongly encouraged in order to design effective instructional programs for students.

Learning Progressions:

The Common Core State Standards in mathematics were built on progressions: narrative documents describing the progression of a topic across a number of grade levels, informed both by research on children's cognitive development and by the logical structure of mathematics. The progressions documents can explain why standards are sequenced the way they are, point out cognitive difficulties and pedagogical solutions, and give more detail on particularly knotty areas of the mathematics. This would be useful in teacher preparation and professional development, organizing curriculum, and writing textbooks.

Standards for Mathematical Practice:

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

Los Angeles Unified School District
Office of Curriculum Instruction and School Support
2015-2016 Common Core Geometry Curriculum Map

The Mathematics Curriculum Map is a living document—it is neither set in stone for all time nor is it perfect. Teachers and other users are encouraged to provide on-going feedback as to its accuracy, usability, and content. Please go to math.lausd.net and share your comments and suggestions. Your participation in making this instructional guide a meaningful and useful tool for all is needed and appreciated.

The grade level Common Core State Standards-aligned Mathematics Curriculum Maps of the courses in this 2014 edition of the *CCSS Mathematics Curriculum Map* are the result of the collective expertise of the LAUSD Secondary Mathematics Team.

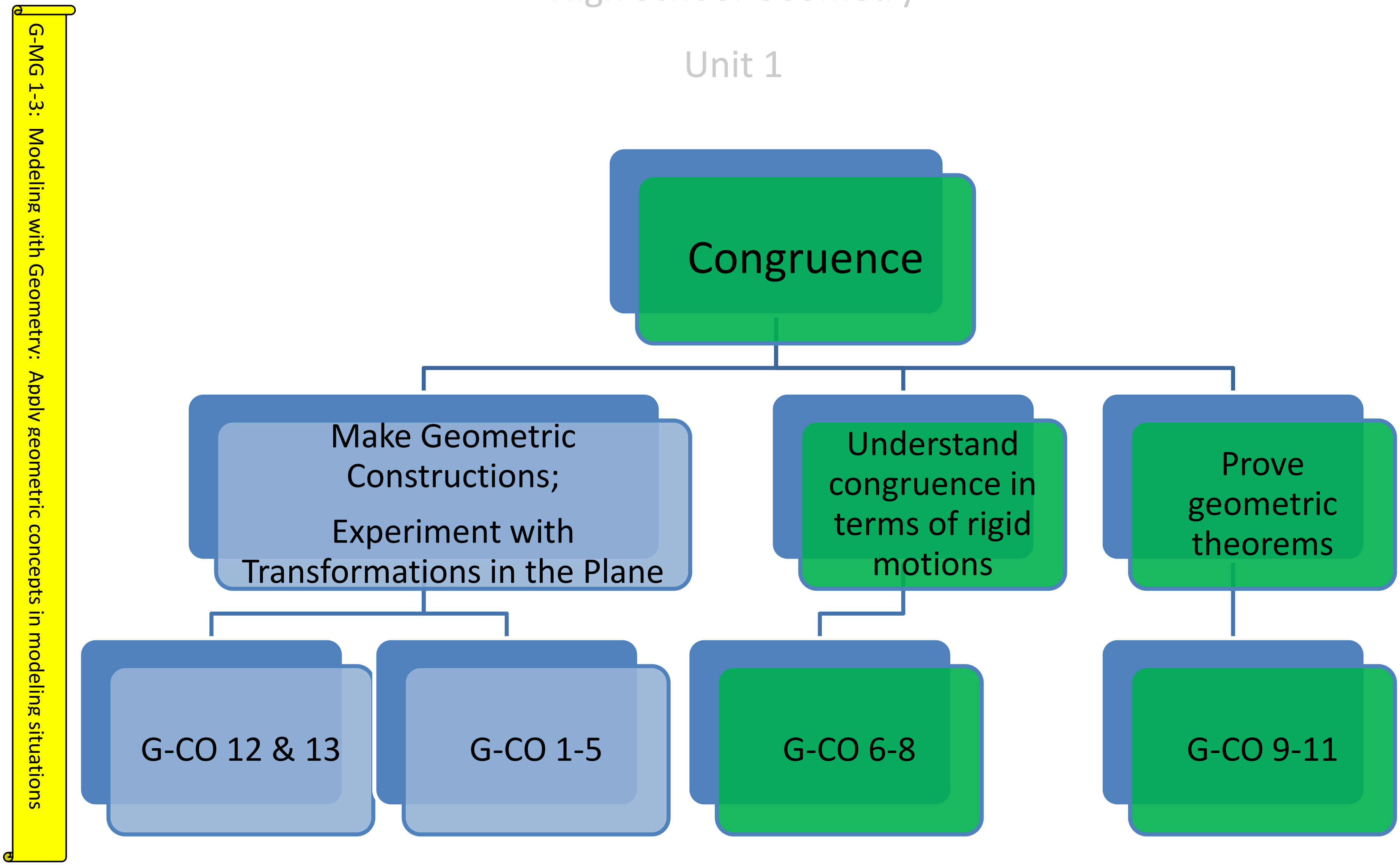
The District extends its gratitude to the following Geometry curriculum map development team:

Elisa Rose, Andres Flores, Lisa Usher, Jane Berman, Reginald Brookens, Jared Dupree, Barbara Jacobs-Ledbetter, Susan Mussack, Kimberly Montsinger, Julia Keiper, Norma Grimaldo-Ramirez, Marla Mattenson, Roxanne Klarin, Roslyn Lewis-Chambers, Diana Tabbara, Kekai Bryant-Williams, Aris Biegler, Norma Alvarez, and Helen Choi.

This document was developed under the auspices of the Chief Academic Officer of the Office of Curriculum, Instruction and School Support, Gerardo Loera. Particular gratitude is extended to Caroline Piangerelli, Lisa Ward, Laura Cervantes, and Philip Ogbuehi, who coordinated the 2015 edition initiative under the guidance of Angel Barrett, Executive Director of the K-12 Instruction.

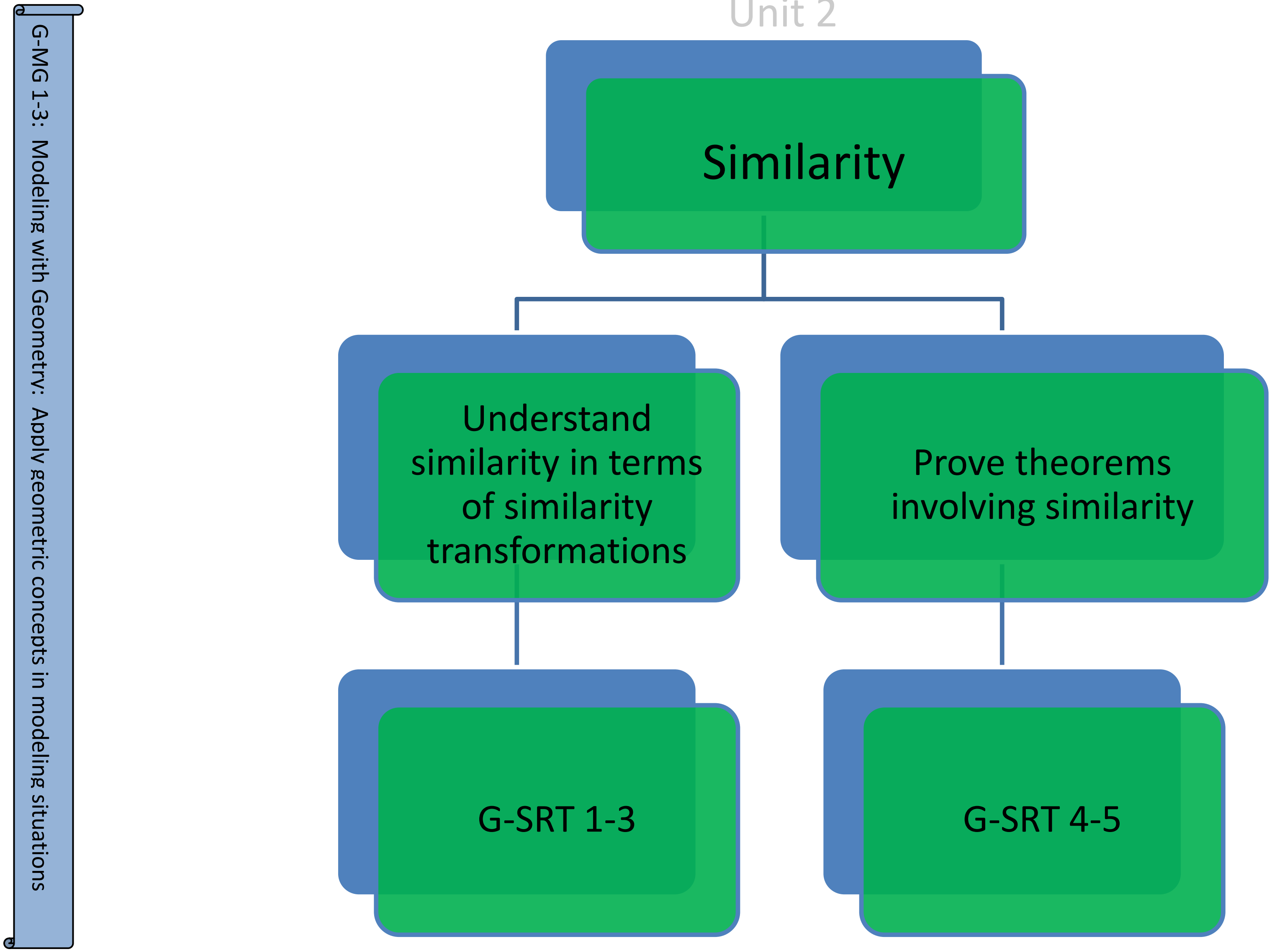
High School Geometry

Unit 1



Key: Major Clusters; Supporting Clusters; Additional Clusters

High School Geometry
Unit 2



Key: Major Clusters; Supporting Clusters; Additional Clusters

High School Geometry
Unit 3

G-MG 1-3: Modeling with Geometry: Apply geometric concepts in modeling situations

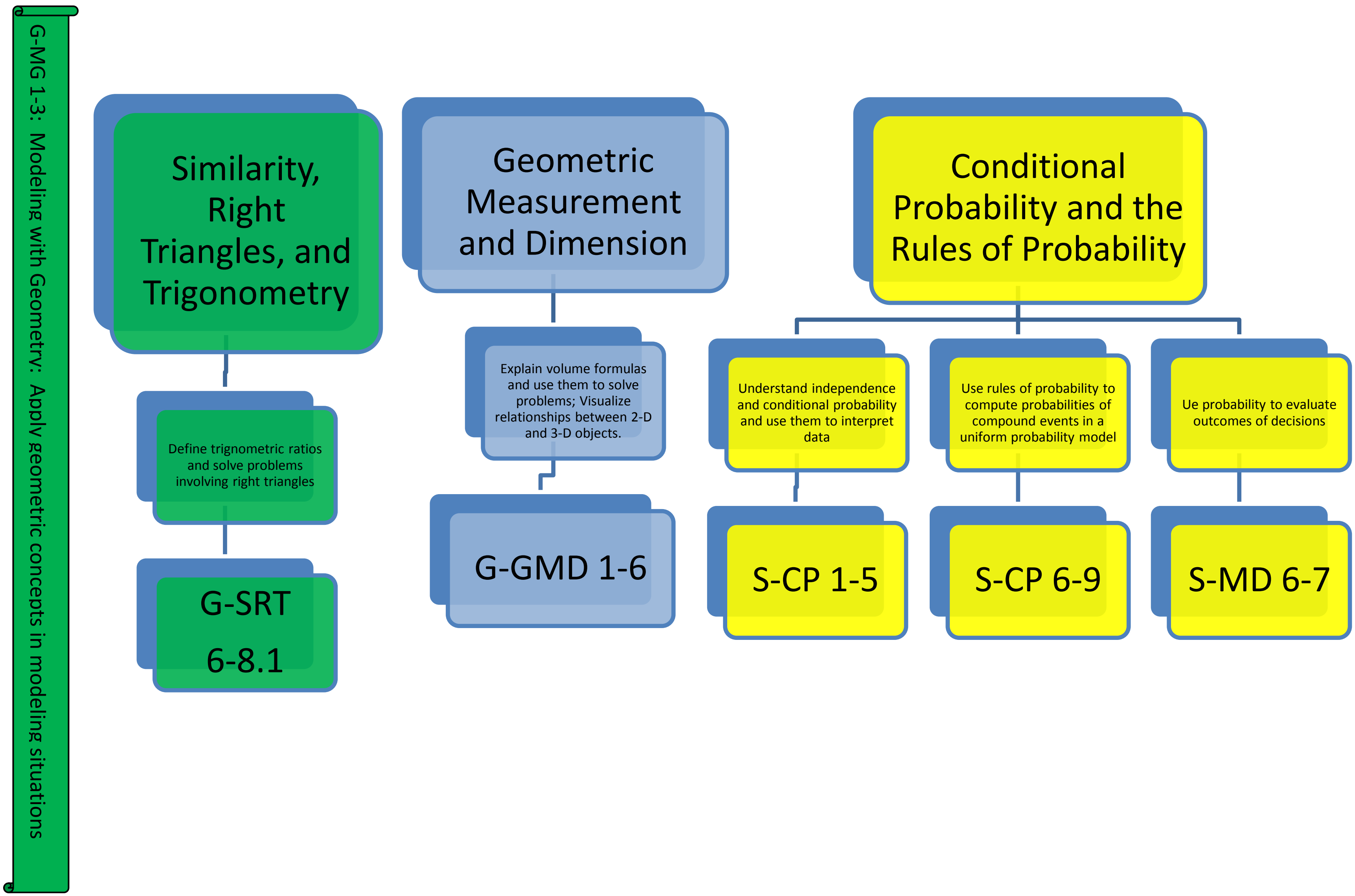
Expressing Geometric Properties with Equations; Circles

- Use coordinates to prove simple geometric theorems algebraically
  - GPE 4-7
- Understand and prove theorems about circles; Find arc lengths and areas of sectors of circles.
  - G-C 1-5
- Translate between the geometric description and the equation for a conic section
  - GPE 1-2

Key: Major Clusters; Supporting Clusters; Additional Clusters

High School Geometry

Unit 4



Key: Major Clusters; Supporting Clusters; Additional Clusters

High School Geometry – Unit 1

Develop the ideas of congruence through constructions and transformations

Critical Area: In this Unit the notion of two-dimensional shapes as part of a generic plane (the Euclidean Plane) and exploration of transformations of this plane as a way to determine whether two shapes are congruent or similar are formalized. Students use transformations to prove geometric theorems. The definition of congruence in terms of rigid motions provides a broad understanding of this notion, and students explore the consequences of this definition in terms of congruence criteria and proofs of geometric theorems. Students develop the ideas of congruence and similarity through transformations.

CLUSTERS	COMMON CORE STATE STANDARDS
Make geometric construction *Make a variety of formal geometric constructions using a variety of tools.*	**Geometry - Congruence** **G.CO.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software etc. Copying a segment, copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines including the perpendicular bisector of a line segment; and constructing a line parallel to a give line through a point not on the line. **G.CO.13** Construct an equilateral triangle, a square, a regular hexagon inscribed in a circle.
Experiment with transformations in the plan *Develop precise definitions of geometric figures based on the undefined notions of point, line, distance along a line and distance around a circular arc.* *Experiment with transformations in the plane.*	**Geometry - Congruence** **G.CO.1** Know precise definitions of angle, circle, perpendicular lines, parallel lines, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. **G.CO.2** Represent transformations in the plane using e.g. transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g. translation versus horizontal stretch.) **G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. **G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles perpendicular lines, parallel lines, and line segments. **G.CO.5** Given a geometric figure and a rotation, reflection or translation, draw the transformed figure using e.g. graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
Understand congruence in terms of rigid motions *Use rigid motion to map corresponding parts of congruent triangle onto each other.* *Explain triangle congruence in terms of rigid motions.*	**Geometry - Congruence** **G.CO.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. **G.CO.7** Use definition of congruence in terms of rigid motions to show that two triangles are congruent if

CLUSTERS	COMMON CORE STATE STANDARDS
	and only if corresponding pairs of sides and corresponding pairs of angles are congruent. **G.CO.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow the definition of congruence in terms of rigid motions.
Prove geometric theorems Prove theorems about lines and angles, triangles; and parallelograms.	**Geometry - Congruence** **G.CO.9** Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. **G.CO.10** Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. **G.CO.11** Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent; the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
MATHEMATICAL PRACTICES	
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.** 3. **Construct viable arguments and critique the reasoning of others.** 4. **Model with mathematics.** 5. **Use appropriate tools strategically.** 6. **Attend to precision.** 7. **Look for and make use of structure.** 8. Look for and express regularity in repeated reasoning.	As you begin the year, it is advised that you start with MP1 and MP 3 to set your expectations of your classroom. This will help you and your students become proficient in the use of these practices. All other practices may be evident based on tasks and classroom activities.
LEARNING PROGRESSIONS	

(m)Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.

(s)Supporting/Additional Clusters – designed to support and strengthen areas of major emphasis/expose students to other subjects.

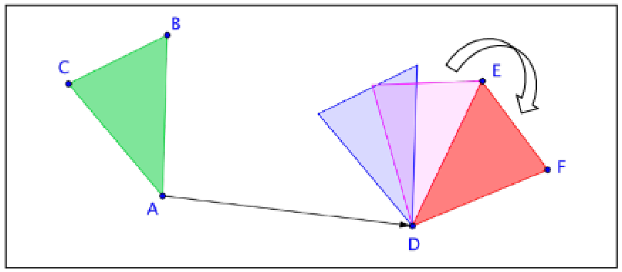
★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• The fundamental tools of classic construction are the	• How do geometric constructions relate to	• alternate Interior Angles

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
compass and the straightedge but, there are many other tools useful for constructions including; string, reflective devices, protractor, and geometric software. • Geometric construction is a visual representation of geometric principals and develops a deeper understanding of the spatial relationships between pairs of figures and their elements. • Transformations include a variety of motions that take a set of points in the plane as input and gives us other points as output. • There are rigid transformations that preserve distance and angles and non-rigid transformations that do not. • The properties of transformations that are rigid motion can be used to identify and prove congruence of figures in a plane. • Constructing a viable argument using the precise vocabulary of transformations and congruence to prove geometric theorems in a variety of formats is important to Geometry proof.	geometric to geometric reasoning and proof? • What are the justifications that can be used to guide geometric constructions? • What are the criteria that can be used by a geometry student to select the most appropriate tools and software for geometric constructions? • What are the similarities and differences among the various transformations and how can they be grouped as either rigid or non-rigid? • How can the properties of rigid motion be used to prove that two triangles are congruent (ASA, SAS, SSS)? • What are the various pathways to create a valid proof for theorems about lines, angles, triangles congruence and parallelograms?	• compass • congruence • construction • corresponding • distance • equilateral Triangle • isosceles Triangle • mapping • median • midpoint • non-rigid motion • parallel Lines • parallelogram • perpendicular Lines • protractor • reflection • rigid Motion • rotation • straightedge • transformations • translation • vertical Angles

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
LAUSD Adopted Textbooks and Programs • Big Ideas Learning - Houghton Mifflin Harcourt, 2015: Big Ideas Geometry • College Preparatory Mathematics, 2013: Core Connections, Geometry • The College Board, 2014:Springboard Geometry **Materials:** For Students: compass, protractor, straight-edge, string, reflective devices, tracing paper, graph paper and geometric software.	Engage students to investigate more closely the definition that shapes are congruent when they "have the same size and shape." Earlier, students experimented with transformations in the plane, but now, students build more precise definitions for the rigid motions (rotation, reflection, and translation) based on previously defined and understood terms, such as point, line, between, angle, circle, perpendicular, etc. (G-CO.1,3,4). Help students strengthen their understanding of these definitions by transforming figures using patty paper,	**Formative Assessment** PARCC - http://www.parcconline.org/samples/mathematics/high-school-mathematics http://www.parcconline.org/sites/parcc/files/PARCC_SampleItems_Mathematics_HSGeoMathIIIGeometricConnection_081913_Final_0.pdf

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
For instruction: Document camera, LCD projector, screen **Websites**: Math Open Reference http://mathopenref.com/tocs/constructionstoc.html (online resource that illustrates how to generate constructions) Math is Fun http://www.mathsisfun.com/geometry/constructions.html H-G.CO.12, 13 Manga High http://www.mangahigh.com/en_us/games/transtar **Engage New York** http://www.engageny.org/sites/default/files/resource/attachments/geometry-m1-teacher-materials.pdf	transparencies, or geometry software, (G-CO.2, 3,5, MP.). Transformations should be investigated both in a general plane as well as on a coordinate system especially when explicitly describing transformations using precise names of points, translation vectors, and lines of symmetry or reflection. **Concrete Models** – Students make use of visual tools for representing geometric figures, such as simple patty paper or transparencies, graph paper, calculators, reflective devices, dynamic geometry software, or other manipulatives as they work through transformations. Have students show using rigid motions that congruent triangles have congruent corresponding parts, and that, conversely, if the corresponding parts of two triangles are congruent, then there is a sequence of rigid motions that takes one triangle to the other. For example:  Illustration of the reasoning that corresponding parts being congruent implies triangle congruence, in which point A is translated to D, the resulting image of Δ ABC is rotated so as to place B onto E, and finally, the image is then reflected along line segment DE to match point C to F. **Geometry Construction** – Students use a variety of tools and methods to make formal geometric constructions, such as: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a	**LAUSD Assessments** The district will be using the SMARTER Balanced Interim Assessments. Teachers would use the Interim Assessment Blocks (IAB) to monitor the progress of students. Each IAB can be given twice to show growth over time. **State Assessments** California will be administering the SMARTER Balance Assessment as the end of course for grades 3-8 and 11. There is no assessment for Algebra 1. The 11th grade assessment will include ítems from Algebra 1, Geometry, and Algebra 2 standards. For examples, visit the SMARTER Balance Assessment at: http://www.smarterbalanced.org/ Sample Smarter Balanced Items: http://sampleitems.smarterbalanced.org/itempreview/sbac/index.htm

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
	line parallel to a given line through a point not on the line. Teachers should use a variety of strategies for engaging students in understanding and writing proofs, including: using ample pictures to demonstrate results and generate strategies; using patty paper, transparencies, or dynamic geometry software to explore the steps in a proof; creating flow charts and other organizational diagrams for outlining a proof; and writing step-by-step or paragraph formats for the completed proof (MP.5).	

LANGUAGE GOALS for **low achieving, high achieving, students with disabilities and English Language Learners**
• Students justify congruency statements using key vocabulary, such as: mapping, translation, reflection, rotation, rigid motion, and congruence. • Students describe their understanding of a construction using key vocabulary, such as: bisect an angle, perpendicular bisector, and parallel lines. • Students identify words in word problems that help them formulate arguments and evaluate arguments to make specific claims about congruence; they will use the sentence starter, "The words _________ and _________lead me to believe…" • Students compare two geometric shapes using comparative adjectives. • Students will compare transformations in the plane and describe their changes using academic language and complete sentences.

PERFORMANCE TASKS
Circles in Triangles http://map.mathshell.org/materials/tasks.php?taskid=256#task256 http://map.mathshell.org/materials/tasks.php?taskid=258#task258

DIFFERENTIATION📖		
UDL/FRONT LOADING	ACCELERATION	INTERVENTION
Know the basic properties of the different types of triangles (equilateral, equiangular, isosceles, right angle, scalene, obtuse, acute). Work with construction tools: drawing circles, measuring with compass, drawing lines. Know how to name angles, points, lines, rays,	Students can learn to prove and develop theorems for transformations that are not on the coordinate plane using conditional statements in their explanations. Condense the units of circles and transformations together; use the properties of circles to determine points of rotation.	Model and review constructions (online resources). Include and use vocabulary lists with visual aids. Use heterogeneous groups for peer assistance and modeling.
FRONT LOADING	ACCELERATION	INTERVENTION
segments and length. Know distance, midpoint and slope formulae. Know how to plot points.	Combine dilations and similarity, showing parallelism, angle congruence in dilated figures and the definition of dilation to prove shapes are similar through AA. Make use of isosceles triangle and third angles theorems.	

References:

1. National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards (Mathematics)*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.
2. McCallum, W., Zimba, J., Daro, P. (2011, December 26 Draft). *Progressions for the Common Core State Standards in Mathematics*. Cathy Kessel (Ed.). Retrieved from http://ime.math.arizona.edu/progressions/#committee.
3. Engage NY. (2012). New York Common Core Mathematics Curriculum. Retrieved from http://www.engageny.org/resource/high-school-geometry.
4. Mathematics Assessment Resource Service, University of Nottingham. (2007 - 2012). Mathematics Assessment Project. Retrieved from http://map.mathshell.org/materials/index.php.
5. Smarter Balanced Assessment Consortium. (2012). Smarter Balanced Assessments. Retrieved from http://www.smarterbalanced.org/.
6. Partnership for Assessment of Readiness for College and Career. (2012). PARCC Assessments. Retrieved from http://www.parcconline.org/parcc-assessment.
7. California Department of Education. (2013). Draft Mathematics Framework Chapters. Retrieved from http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp.
8. National Council of Teachers of Mathematics (NCTM) Illuminations. (2013). Retrieved from http://illuminations.nctm.org/Weblinks.aspx.
9. The University of Arizona. (2011-12). Progressions Documents for the Common Core Math Standards. Retrieved from http://ime.math.arizona.edu/progressions.

   Artzt, Alice F., and Claire M. Newman. How to Use Cooperative Learning in the Mathematics Classroom. Reston, VA: NCTM, 1997.

Geometry – UNIT 2
Similarity, Right Triangles, and Trigonometry

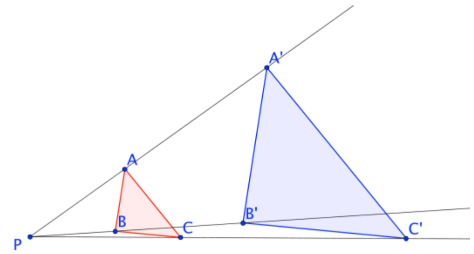
Critical Area: Students investigate triangles and decide when they are similar. A more precise mathematical definition of similarity is given; the new definition taken for two objects being similar is that there is a sequence of similarity transformations that maps one exactly onto the other. Students explore the consequences of two triangles being similar: that they have congruent angles and that their side lengths are in the same proportion. Students prove the Pythagorean Theorem using triangle similarity.

CLUSTERS	COMMON CORE STATE STANDARDS
Understand similarity in terms of similarity transformations	**Geometry - Similarity, Right Triangles, and Trigonometry** **G-SRT.1**. Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. **G-SRT.2**. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **G-SRT.3**. Use the properties of similarity transformations to establish the Angle-Angle (AA) criterion for two triangles to be similar.
Prove theorems involving similarity **Apply geometric concepts in modeling situations**	**Geometry - Similarity, Right Triangles, and Trigonometry** **G-SRT.4**. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. **G-SRT.5**. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures Supporting clusters: **G-MG 1-3**: Modeling with Geometry: Apply geometric concepts in modeling situations
MATHEMATICAL PRACTICES	
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.** 3. **Construct viable arguments and critique the reasoning of others.** 4. **Model with mathematics.** 5. **Use appropriate tools strategically.** 6. **Attend to precision.**	Emphasize Mathematical Practices 1, 2, 3, 4, 4, 5, and 6 in this unit.

7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• Sequence of similarity transformation of two objects that maps one exactly onto the other is defined. • Similarity of two objects using their given ratio by a scale factor is proved; such as: using the dilation of a line segment in ratio given by the scale factor. • Similar triangles have corresponding pairs of angles and proportional pairs of sides (AA, SAS, SSS). • Prove Theorems about triangles; such as "a line parallel to one side of a triangle divides the other two proportionately and conversely." • Triangle similarity is used to prove the Pythagorean Theorem. • Congruence and similarity criteria for triangles are used to solve problems and prove relationships of geometric figures.	• What is the difference between similarity and congruence? • How can you show that it is not possible to prove similarity by showing three angles in proportion to one another? • How do you construct a viable argument for congruency and/or similarity of two triangles? • How do you construct a viable argument for the similarity of geometric figures? • Are all congruent triangles similar and is the converse true also?	• congruency • corresponding • criterion • derive • dilation • dilation of scale factor • parallel lines • proportionality • reflection • rigid motion • rotation • scale factor • sequence • similar • similarity transformation • transversal • triangle relationships

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
LAUSD Adopted Textbooks and Programs • Big Ideas Learning - Houghton Mifflin Harcourt, 2015: Big Ideas Geometry • College Preparatory Mathematics, 2013: Core Connections, Geometry • The College Board, 2014:Springboard Geometry **Mathematics Assessment Project (MARS Tasks)** Hopwell Geometry – G.SRT.5 http://map.mathshell.org/materials/download.ph	• Provide students the opportunities to experiment with dilations and determine how they affect planar objects. Have them explore the properties of dilations in more detail and develop an understanding of the notion of scale factor (G-SRT.1). Students first make sense of the definition of a dilation of scale factor $k>0$ with center P as the transformation that moves a point A along the ray $\overrightarrow{PA}$ to a new point A', so that $\lvert\overrightarrow{PA'}\rvert = k \cdot \lvert\overrightarrow{PA}\rvert$ • For example, students apply the dilation of scale factor 2.5 with center P to the points A, B, and C	**Formative Assessment**

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
p?fileid=499 Inscribing and Circumscribing Right Triangles – G.SRT: http://map.mathshell.org/materials/lessons.php?taskid=403&subpage=problem **Illustrative Mathematics** Similar Triangles : G-SRT.3 http://www.illustrativemathematics.org/illustrations/1422 Pythagorean Theorem : G-SRT.4 http://www.illustrativemathematics.org/illustrations/1568 Joining two midpoints of sides of a triangle : G-SRT.4 http://www.illustrativemathematics.org/illustrations/1095	illustrated using a ruler. Once they've done so, they consider the two triangles $\Delta\ ABC$ and $\Delta A'\ B'C'$. What they discover is that the lengths of the corresponding sides of the triangles have the same ratio as dictated by the scale factor. Students learn that parallel lines are taken to parallel lines by dilations; thus corresponding segments of ΔABC and ΔA'B'C' are parallel. After students have proved results about parallel lines intersected by a transversal, they can deduce that the angles of the triangles are congruent.  Through experimentation, they see that the congruence of corresponding angles is a necessary and sufficient condition for the triangles to be similar, leading to the AA criterion for triangle similarity. (G.SRT.3.) For a simple investigation, students can observe how the distance at which a projector is placed from a screen affects the size of the image on the screen. (MP.4) • Have students use geometric shapes, their measures, and their properties to describe objects including two- and three-dimensional shapes.	**LAUSD Assessments** The district will be using the SMARTER Balanced Interim Assessments. Teachers would use the Interim Assessment Blocks (IAB) to monitor the progress of students. Each IAB can be given twice to show growth over time. **State Assessments** California will be administering the SMARTER Balance Assessment as the end of course for grades 3-8 and 11. There is no assessment for Algebra 1. The 11th grade assessment will include ítems from Algebra 1, Geometry, and Algebra 2 standards. For examples, visit the SMARTER Balance Assessment at: http://www.smarterbalanced.org/ Sample Smarter Balanced Items: http://sampleitems.smarterbalanced.org/itempreview/sbac/index.htm

LANGUAGE GOALS for **low achieving, high achieving, students with disabilities and English Language Learners**
• Students will be able to articulate orally and in writing the differences between similarity and congruence. • Students will be able to affirm the veracity of mathematical statements. • Students will be able to articulate the process of constructing viable arguments. • Students will be able to describe in writing the definition of similarity in terms of similarity transformations and decide if they are similar. • Students will explain in writing and orally similarity transformations and the meaning of similarity for triangles as the equality of all corresponding pairs of angles.

PERFORMANCE TASKS
Illustrative Mathematics Dilating a Line : G-SRT.1 http://www.illustrativemathematics.org/illustrations/602 Are they Similar ?: G-SRT.2 http://www.illustrativemathematics.org/illustrations/603 Folding a Square into Thirds : G-SRT.5 http://www.illustrativemathematics.org/illustrations/1572 **LAUSD Concept Lessons** – http://math.lausd.net Squaring Triangles **Mathematics Assessment Project (MARS Tasks):** Geometry Problems: Circles and Triangles – G-SRT http://map.mathshell.org/materials/lessons.php?taskid=222#task222 **Inscribing and Circumscribing Right Triangles -** http://map.mathshell.org/materials/lessons.php?taskid=403&subpage=problem **Modeling: Rolling Cups-** http://map.mathshell.org/materials/lessons.php?taskid=428&subpage=problem Solving Geometry Problems: Floodlights – G-SRT.5, G-MG.1-3 http://map.mathshell.org/materials/lessons.php?taskid=429&subpage=problem Analyzing Congruence Proofs – G-CO.6-8 http://map.mathshell.org/materials/lessons.php?taskid=452&subpage=concept **Calculating Volumes of Compound Objects – G-MD** http://map.mathshell.org/materials/lessons.php?taskid=216&subpage=concept Proofs of the Pythagorean Theorem http://map.mathshell.org/materials/lessons.php?taskid=419&subpage=concept

DIFFERENTIATION🕮		
UDL/FRONT LOADING	ACCELERATION	INTERVENTION
Prerequisites: • •Assessment tasks can be given a day prior in class or as homework to find out the difficulties students have prior to the lessons. Clarify the objectives in student friendly language and communicate the learning expectations by the end of the concept development tasks to lower the	• Advanced students should have access to bank of more challenging problems for extension • Gifted and advanced student can use alternate projects, to meet their unique needs. • Use of technology and software to enhance student learning and explore further.	Multiple entry points for problems should be planned and taught in each lesson. When the lesson is reviewed or retaught, use a different entry point or a different method. Illicit more information about students' misconceptions or misunderstandings before choosing

students' anxiety.		or recommending strategies aligned with math goals and students' abilities. Use higher order questions and effective questioning techniques to enhance learning; analyze skills and evaluate students' understanding. To increase active participation, students should be expected to work collaboratively to promote authentic conversation, increase opportunities for asking questions, and peers support. Use visual tools, academic language, graphic organizers, manipulatives, and engaging and real world examples. Make clear connections to prior grade concepts See "Common Issues" of each Mars Tasks

References:

1. National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards (Mathematics)*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.
2. McCallum, W., Zimba, J., Daro, P. (2011, December 26 Draft). *Progressions for the Common Core State Standards in Mathematics*. Cathy Kessel (Ed.). Retrieved from http://ime.math.arizona.edu/progressions/#committee.
3. Engage NY. (2012). New York Common Core Mathematics Curriculum. Retrieved from http://engageny.org/sites/default/files/resource/attachments/a-story-of-ratios-a-curriculum-overview-for-grades-6-8.pdf.
4. Mathematics Assessment Resource Service, University of Nottingham. (2007 - 2012). Mathematics Assessment Project. Retrieved from http://map.mathshell.org/materials/index.php.
5. Smarter Balanced Assessment Consortium. (2012). Smarter Balanced Assessments. Retrieved from http://www.smarterbalanced.org/.
6. Partnership for Assessment of Readiness for College and Career. (2012). PARCC Assessments. Retrieved from http://www.parcconline.org/parcc-assessment.
7. California Department of Education. (2013). Draft Mathematics Framework Chapters. Retrieved from http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp.
8. National Council of Teachers of Mathematics (NCTM) Illuminations. (2013). Retrieved from http://illuminations.nctm.org/Weblinks.aspx.
9. The University of Arizona. (2011-12). Progressions Documents for the Common Core Math Standards. Retrieved from http://ime.math.arizona.edu/progressions.

High School Geometry – Unit 3

Express Geometric Properties with Equations; Extend Similarity to Circles

Critical Area: Students investigate triangles and decide when they are similar; with this newfound knowledge and their prior understanding of proportional relationships, they define trigonometric ratios and solve problems using right triangles. They investigate circles and prove theorems about them. Connecting to their prior experience with the coordinate plane, they prove geometric theorems using coordinates and describe shapes with equations. Students extend their knowledge of area and volume formulas to those for circles, cylinders and other rounded shapes. They prove theorems, both with and without the use of coordinates.

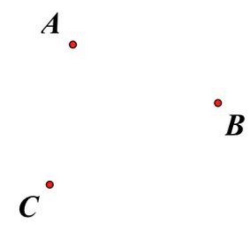
CLUSTERS	COMMON CORE STATE STANDARDS
Use coordinates to prove simple geometric theorems algebraically	**Geometry - Expressing Geometric Properties with Equations** **G.GPE.4**. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1,* $\sqrt{3}$ *) lies on the circle centered at the origin and containing the point (0, 2).* **G.GPE.5**. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). **G.GPE.6**. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. **G.GPE.7**. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. ★
Understand and apply theorems about circles **Find arc lengths and areas of sectors of circles**	**Geometry - Circles** **G.C.1**. Prove that all circles are similar. **G.C.2.** Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* **G.C.3.** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. **G.C.5.** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. **Convert between degrees and radians. CA**
Translate between the geometric description and the equation for a conic section	**Geometry - Expressing Geometric Properties with Equations** **G.GPE.1**. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **G.GPE.2**. Derive the equation of a parabola given a focus and directrix.
MATHEMATICAL PRACTICES	

1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.** 3. **Construct viable arguments and critique the arguments of others.** 4. **Model with mathematics.** 5. **Use appropriate tools strategically.** 6. **Attend to precision.** 7. **Look for and make use of structure.** 8. Look for and express regularity in repeated reasoning.	As you begin this unit, it is advised that you start with MP1 and MP 3 to set up your expectations of your classroom. This will help you and your students become proficient in the use of these practices. Emphasize Mathematical Practices 1, 2, 3, 4, 5, 6, and 7 in this unit.
LEARNING PROGRESSIONS	

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• Express a geometric relationship algebraically (e.g. the Pythagorean Theorem) to new situations such as deriving equation of a circle using the distance formula or deriving the equation of a parabola in terms of focus and directrix. • Right triangle and triangle similarity can be applied to geometric and algebraic theorems to find coordinates of a point on a line given proportion of segments on the line. • Justify algebraically the relationships between slopes of parallel and perpendicular lines as they can be established through proof. • The algebraic representation of a geometric problem can be used to prove theorems in a coordinate plane. • The concept of similarity as it relates to circles can be extended with proof. • Relationships between angles, radii and chords will be investigated. • Similarities will be applied to derive an arc length and a sector area.	• Given coordinate plane information, can we prove (or disprove) geometric relationships (e.g. given the vertices, disprove the assertion that ABCD is a rhombus; or that a given point lies on a circle)? • What is always true about the slopes of perpendicular (or, parallel) lines, and how can a proof be written to exemplify this? • How might we use “constant of proportionality” to define radian measure? • How can we write the equation for a circle or parabola? • How can algebraic representation of a geometric problem be used to prove theorems in coordinate plane? • How can the relationships between angles, radii, and cords be investigated?	• arc • circumscribed • focus • derive • directrix • focus • inscribed • intersect • parallel line • perpendicular line • polygon • Pythagorean Theorem • sector • similar • tangent • vector (directed line segment)

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
LAUSD Adopted Textbooks and Programs • Big Ideas Learning - Houghton Mifflin Harcourt, 2015: Big Ideas Geometry • College Preparatory Mathematics, 2013: Core Connections, Geometry • The College Board, 2014:Springboard Geometry **Materials**: • Compass, straight-edge, graph paper, reflective surface, protractor, tracing paper, scissors, tape. • Geometer's Sketchpad or other software. Geogebra Software **Illustrative Mathematics** Right triangles inscribed in circles II: G.C.2a **http://www.illustrativemathematics.org/illustrati**	Teachers are encouraged to use a variety of strategies for engaging students in understanding and writing proofs, including: using ample pictures or diagrams to demonstrate results and generate strategies; using patty paper, transparencies, or dynamic geometry software to explore the steps in a proof; creating flow charts and other organizational diagrams for outlining a proof; and writing step-by-step or paragraph formats for the completed proof (MP.5). Design an activity where students extend their understanding of the usefulness of similarity transformations through investigating circles (G-C.1). For instance, students can reason that any two circles are similar by describing precisely how to transform one onto the other, as the example	**Formative Assessments** – include checking for understanding using dry-erase boards, exit tickets such as the following activity: You have been asked to place a fire hydrant so that it is an equal distance form three locations indicated on the following map. Show how to fold your paper to physically construct this point as an intersection of two creases (http://www.illustrativemathematics.org/illustrations/508). 

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
ons/1093 Inscribing a triangle in a circle : G.C.**3a** **http://www.illustrativemathematics.org/illustrations/1013** Two Wheels and a Belt : G.C. B **http://www.illustrativemathematics.org/illustrations/621** Equal Area Triangles on the Same Base II : G.GPE.5b **http://www.illustrativemathematics.org/illustrations/1348**	illustrates with two specific circles. **Example.** Students can show that the two circles C and D given by the equations below are similar. $C: (x-1)^2 + (y-4)^2 = 9$ $D: (x+2)^2 + (y-1)^2 = 25$ **Solution.** Since the centers of the circles are $(1, 4)$ and $(-2, 1)$, respectively, we first translate the center of circle C to the center of circle D using the translation $T(x, y) = (x-3, y-3)$. Finally, since the radius of circle C is 3 and the radius of circle D is 5, we dilate from the point $(-2, 1)$ by a scale factor of $5/3$. **Geometry Construction** – Students use a variety of tools and methods to make formal geometric constructions, such as: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.	LAUSD Assessment District assessments are under development. More information to come soon. State Assessments California will be administering the SMARTER Balance Assessment as the end of course for grades 3-8 and 11. There is no assessment for Algebra 1. The 11th grade assessment will include ítems from Algebra 1, Geometry, and Algebra 2 standards. For examples, visit the SMARTER Balance Assessment at: SBAC - http://www.smarterbalanced.org/

LANGUAGE GOALS for **low achieving, high achieving, students with disabilities and English Language Learners**

- Students organize their math thinking and use precise language of mathematics to describe their findings.
- Students communicate their math thinking clearly orally and by writing with their peers and the teacher using academic vocabulary.
- Students will describe in writing the characteristics between inscribed angles and central angles using key vocabulary.
- Students will identify words in word problems that help them formulate arguments; they will use the sentence starter, "The words _________ and _________ lead me to believe…"
- Students will evaluate arguments and make specific claims about slopes of perpendicular and parallel lines.
- Students will compare circles and describe its' similarity using complete sentences and academic language (dilation, ratio).
- Students will make predictions about a problem using predicting verbs and give a reason for their prediction using supporting vocabulary.

PERFORMANCE TASKS

Illustrative Mathematics

Right triangles inscribed in circles I: G.C.2 **http://www.illustrativemathematics.org/illustrations/1091**

Tangent Lines and the Radius of a Circle : G.C.2a, 9 **http://www.illustrativemathematics.org/illustrations/963**

Locating Warehouse : G.C.3a, G.CO.13 **http://www.illustrativemathematics.org/illustrations/507**

Tangent to a circle from a point : G.C.4a **http://www.illustrativemathematics.org/illustrations/1096**

A Midpoint Miracle : G.GPE.4b, 5b **http://www.illustrativemathematics.org/illustrations/605**

Slopes and Circles : G.GPE.1 **http://www.illustrativemathematics.org/illustrations/479**

Explaining the Equation for a Circle : G.GPE.1 **http://www.illustrativemathematics.org/illustrations/1425**

Finding Triangle Coordinates : G.GPE.6, G.SRT.**5** **http://www.illustrativemathematics.org/illustrations/1685**

PERFORMANCE TASKS

LAUSD Concept Lessons

http://math.lausd.net

The Bermuda Triangle

Awesome Amanda

Mathematics Assessment Project (MARS Tasks):

Inscribing and Circumscribing Right Triangles - G.C.3 http://map.mathshell.org/materials/lessons.php?taskid=403&subpage=problem

Geometry Problems: Circles and Triangles-http://map.mathshell.org/materials/lessons.php?taskid=222&subpage=problem

Finding Equations of Parallel and Perpendicular Lines –G.GPE.5 http://map.mathshell.org/materials/lessons.php?taskid=226&subpage=concept

Sectors of Circles-G.C.5 http://map.mathshell.org/materials/lessons.php?taskid=441&subpage=concept

Equations of Circles 1-G.GPE.1 http://map.mathshell.org/materials/lessons.php?taskid=406&subpage=concept

Equations of Circles 2: G.GPE.3.1; MP 1,7: http://map.mathshell.org/materials/lessons.php?taskid=425#task425		
DIFFERENTIATION📖		
UDL/FRONT LOADING	ACCELERATION	INTERVENTION
Assessment tasks can be given a day prior in class or as homework to determine the difficulties students have prior to the lessons. Clarify the objectives in student-friendly language and communicate the learning expectations by the end of the concept development tasks to lower the anxiety. Provide examples of completing the square. Slope, midpoint, distance formulae. Definition of a circle, review circle formulae from prior grade. Review some of the following depending on your students' strength: the coordinate system, solving algebraic equations and inequalities, Pythagorean Theorem, definition of a Parabola, constructing right triangles, triangle similarity, and slopes of parallel and perpendicular lines	Combine trigonometric ratios with the concepts of 30-60-90, 45-45-90 triangles to have students determine the connection between radian and degree measures and developing the Unit Circle. Engage students to apply parabola to focus a signal from either the concave or convex side of the parabola. Determine the value of pi by using the properties of inscribed and circumscribed squares and hexagons. Neglecting the Curvature of the Earth: http://www.illustrativemathematics.org/illustrations/1345	• Multiple entry points for problems should be planned. When the lesson is reviewed or retaught use a different entry point or a different method. • Inquire about students' misconception or misunderstanding before choosing or recommending strategies aligned with math goals and students' abilities. • Use higher order questions and effective questioning techniques to enhance learning, analyzing skills and evaluation. • To increase active participation, students should be expected to work collaboratively to help language learners to lower anxiety, promote authentic conversation, create opportunities for asking questions, and support peers and teachers. • Use visual tools, academic language, graphic organizers, manipulatives, and engaging real world examples to develop interest. • Make clear connections to prior grade concepts of slope, parallel and perpendicular lines, proportional relationship between lengths, what students know about angles, lines. • Explain to students what the markings on diagrams and constructions mean in plain English and in the language of geometry. • Use construction lines to help understand the diagrams. • Name lines, rays, segments. • See Common Issues of each Mars Task

References:

1. National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards (Mathematics)*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.
2. McCallum, W., Zimba, J., Daro, P. (2011, December 26 Draft). *Progressions for the Common Core State Standards in Mathematics*. Cathy Kessel (Ed.). Retrieved from http://ime.math.arizona.edu/progressions/#committee.
3. Engage NY. (2012). New York Common Core Mathematics Curriculum. Retrieved from http://www.engageny.org/resource/geometry-module-1.
4. Mathematics Assessment Resource Service, University of Nottingham. (2007 - 2012). Mathematics Assessment Project. Retrieved from http://map.mathshell.org/materials/index.php.
5. Smarter Balanced Assessment Consortium. (2012). Smarter Balanced Assessments. Retrieved from http://www.smarterbalanced.org/.
6. Partnership for Assessment of Readiness for College and Career. (2012). PARCC Assessments. Retrieved from http://www.parcconline.org/parcc-assessment.
7. California Department of Education. (2013). Draft Mathematics Framework Chapters. Retrieved from http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp.
8. National Council of Teachers of Mathematics (NCTM) Illuminations. (2013). Retrieved from http://illuminations.nctm.org/Weblinks.aspx.
9. The University of Arizona. (2011-12). Progressions Documents for the Common Core Math Standards. Retrieved from http://ime.math.arizona.edu/progressions.
10. Artzt, Alice F., and Claire M. Newman. How to Use Cooperative Learning in the Mathematics Classroom. Reston, VA: NCTM, 1997.

Critical Area: Students explore probability concepts and use probability in real-world situations. They continue their development of statistics and probability, students investigate probability concepts in precise terms, including the independence of events and conditional probability. They explore right triangle trigonometry, and circles and parabolas. Throughout the course, Mathematical Practice 3, "Construct viable arguments and critique the reasoning of others," plays a predominant role. Students advance their knowledge of right triangle trigonometry by applying trigonometric ratios in non-right triangles.

CLUSTERS	COMMON CORE STATE STANDARDS
Define trigonometric ratios and solve problems involving right triangles.	**Geometry - Similarity, Right Triangles, and Trigonometry** **G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **G.SRT.7** Explain and use the relationship between the sine and cosine of complementary angles. **G.SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **G.SRT.8.1** Derive and use the trigonometric ratios for special right triangles (30°,60°,90°and 45°,45°,90°). CA
Explain volume formulas and use them to solve problems **Visualize relationships between two-dimensional and three-dimensional objects**.	**Geometric Measurement and Dimension** **G.GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* **G.GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. **G.GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. **G.GMD.5** Know that the effect of a scale factor k greater than zero on length, area, and volume is to multiply each by k, k^2, and k^3, respectively; determine length, area and volume measures using scale factors. CA **G.GMD.6** Verify experimentally that in a triangle, angles opposite longer sides are larger, sides opposite larger angles are longer, and the sum of any two side lengths is greater than the remaining side length;

	apply these relationships to solve real-world and mathematical problems. CA
Understand independence and conditional probability and use them to interpret data (Link to data from simulations or experiments.)	**Statistics and Probability - Conditional Probability and the Rules of Probability** **S.CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). **S.CP.2** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. **S.CP.3** Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. **S.CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* **S.CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
Use the rules of probability to compute probabilities of compound events in a uniform probability model	**Statistics and Probability - Conditional Probability and the Rules of Probability** **S.CP.6** Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. **S.CP.7** Apply the Addition Rule, *P*(*A* or *B*) = *P*(*A*) + *P*(*B*) – *P*(*A* and *B*), and interpret the answer in terms of the model. **S.CP.8** (+) Apply the general Multiplication Rule in a uniform probability model, *P*(*A* and *B*) = *P*(*A*)*P*(*B*|*A*) = *P*(*B*)*P*(*A*|*B*), and interpret the answer in terms of the model.

	S.CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.
MATHEMATICAL PRACTICES	
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.** 3. **Construct viable arguments and critique the reasoning of others.** 4. **Model with mathematics.** 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Emphasize Mathematical Practices 1, 2, 3, and 4 in this unit.

LEARNING PROGRESSIONS
Draft High School Progression on Statistics and Probability http://ime.math.arizona.edu/progressions/

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• Understand trigonometric ratios as the relationships between sides and angles in right triangles. • Understand the concept of complementary angles through sine and cosine. • Trigonometric ratios can be derived for special right triangles (30-60-90 and 45-45-90). • Real world problems can be solved using right triangles, trigonometric ratios and the Pythagorean theorem. • The formulas for circumference of a circle, area of a circle; volume of a cylinder, pyramid and cone can be derived using informal reasoning and solve real-world problems involving the volume for cylinders, pyramids, cones and spheres. • The 2-dimensional shapes formed from the cross-	• Based on similarity, how can you connect the concept of side ratios as angle properties to define the three trigonometric ratios? • Using the concept of complementary angles, how are sine and cosine related? • What generalizations can be made about how you can use an equilateral triangle and the Pythagorean Theorem to make generalizations about the 3 trigonometric ratios for special right triangles? • How do you develop the circumference of a circle, area of a circle, volume of a cylinder, pyramid and cone using informal arguments (i.e.	• Addition Rule • Cavalieri's Principle • Circumference • Combination • Complementary • Compound event • Conditional probability • Cone • Cosine • Cross-section • Cylinder • Dependent/independent variable • Derive • Independent probability • Informal Argument

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
sections of a 3-dimensional object and the 3-dimensional object formed by rotating a 2-dimensional object is defined. • A scale factor (k > 0) can affects the length, area and volume of an object. • How angle measures correspond to side lengths in a triangle. (i.e. smallest angle measures are opposite shortest side lengths)is demonstrated. • Triangle Inequality Theorem is verified using measurement. • Conditional probability of A given B as the fraction of B's outcomes that also belong to A, is interpreted and modeled. • Permutations and combinations probabilities of compound events is computed and used to solve problems. • The addition and general multiplication rule can be applied and interpret probability models	paper folding/cutting)? • What generalizations can be made about the cross-sections of 3-dimensional objects and rotations formed from 2-dimensional objects? • How can you use scale factor to determine the length, area, and volume of similar objects? • What generalizations can be made about the relationship between side lengths and angle measures and also the relationship between side lengths? • How can you use triangle inequality theorem and relationship between side lengths and angles measures to solve real-world problems? • How can you explain the concepts of conditional probability and independence in everyday language and everyday situations? • How is permutations and combinations probabilities of compound events used in problem solving? • What interpretation can be made of probabilities' addition and general multiplication rule?	• Multiplication Rule • Outcomes • Permutation • Pyramid • Pythagorean Theorem • Rotation • Scale Factor • Similarity • Sine • Sphere • Tangent • Trigonometric Ratios • Uniform probability • Volume

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
LAUSD Adopted Textbooks and Programs • Big Ideas Learning - Houghton Mifflin Harcourt, 2015: Big Ideas Geometry • College Preparatory Mathematics, 2013: Core Connections, Geometry • The College Board, 2014:Springboard Geometry	Create an informative poster with a T-table explaining when to use permutation or combination formula. Teach students the acronym SOH-CAH-TOA so that they can easily remember the trigonometric ratios.	LAUSD ASSESSMENT The district will be using the SMARTER Balanced Interim Assessments. Teachers would use the Interim Assessment Blocks (IAB) to monitor the progress of students. Each IAB can be given twice to show growth over time.

		STATE ASSESSMENT
Illustrative Mathematics Defining Trigonometric Ratios: G.SRT.6 http://www.illustrativemathematics.org/illustrations/1635 Sine and Cosine of Complementary Angles: G.SRT.7 http://www.illustrativemathematics.org/illustrations/1443 Shortest line segment from a point *P* to a line *L:* G.SRT.8 http://www.illustrativemathematics.org/illustrations/962 Doctor's Appointment: G.GMD.3 http://www.illustrativemathematics.org/illustrations/527 Centerpiece: G.GMD.3 http://www.illustrativemathematics.org/illustrations/514 Area of a circle: G.GMD.1 http://www.illustrativemathematics.org/illustrations/1567 Global Positioning System: G.GMD.4, A.CED.2 Ihttp://www.illustrativemathematics.org/illustrations/1215 Rain and Lightning:S.CP.2,3,5, and 7 http://www.illustrativemathematics.org/illustrations/1112 Lucky Envelopes: S.CP.3 http://www.illustrativemathematics.org/illustrations/944 Random Walk: S.CP.9 http://www.illustrativemathematics.org/illustrations/689 **Illuminations**	Retrieve actual 3-D items (orange, rectangular cake, cheese block etc.) and demonstrate how a cross-section is like cutting these items and how the 2-D shape can be seen after the cut. Post up a problem on probability of compound events on the board for students to solve independently. Using the four walls of the classroom, post different possible solutions that students might arrive at. Students will walk to the wall that has their answer. Each group of students will have to defend their answer by explaining how they got their answer and justify why they are correct.	California will be administering the SMARTER Balance Assessment as the end of course for grades 3-8 and 11. There is no assessment for Algebra 1. The 11th grade assessment will include ítems from Algebra 1, Geometry, and Algebra 2 standards. For examples, visit the SMARTER Balance Assessment at: http://www.smarterbalanced.org/ Sample Smarter Balanced Items: http://sampleitems.smarterbalanced.org/itempreview/sbac/index.htm

Trigonometry for Solving Problems http://illuminations.nctm.org/LessonDetail.aspx?id=L383		

LANGUAGE GOALS for **low achieving, high achieving, students with disabilities and English Language Learners**

- Students will identify words in probability word-problems that will help them solve them using a causative structure like: *The following words (____ and ___) help me solve the problem* or *The words ____ and ____ help me solve the problem.*
- Students will record step-by-step directions for finding the volume of solid figures using transition words like “first,” “second,” “next” and “finally.”
- Students will describe their understanding of a two-way frequency table, using the words *relative*, *percent*, *column/row,* and *dependent/independent events*.
- Students will describe the shapes of two-dimensional cross-sections of three-dimensional objects, and of three-dimensional objects generated by rotations of two-dimensional objects.
- Students will explain and use the relationship between the sine and cosine of complementary angles.
- Students will write a few sentences describing a specific way to use permutation or combination to compute probability of compound events to solve a problem, linking their opinion and reasons using specific words and phrases (such as consequently, and specifically).

PERFORMANCE TASKS

Mathematics Assessment Project

Funsize cans: G-GMD.3 **http://map.mathshell.org/materials/download.php?fileid=756**

Glasses: G-GMD.3 http://map.mathshell.org/materials/tasks.php?taskid=259#task259

Bestsize Cans: G-GMD.3 http://map.mathshell.org/materials/tasks.php?taskid=284#task284

Propane Tanks: G-GMD.3 http://map.mathshell.org/materials/tasks.php?taskid=288#task288

Hopewell Geometry: G-SRT.5 , 6, and 8 http://map.mathshell.org/materials/tasks.php?taskid=127#task127

Illustrative Mathematics

Coins in a circular pattern: G.SRT.8, G.MG http://www.illustrativemathematics.org/illustrations/720

Seven Circles III: G.SRT.8, G.SRT.D, G.MG.1 http://www.illustrativemathematics.org/illustrations/710

Ask a pilot: G.SRT.8 http://www.illustrativemathematics.org/illustrations/1638

Use Cavalieri’s Principle to Compare Aquarium: G.GMD.2, G.MG.1. http://www.illustrativemathematics.org/illustrations/530

Tennis Balls in a Can: G.GMD.4, G.MG.1 http://www.illustrativemathematics.org/illustrations/512 Global Positioning System II: G.GMD.4, G.MG.1 http://www.illustrativemathematics.org/illustrations/1202 The Titanic 1: S.CP.1,4, and 6 http://www.illustrativemathematics.org/illustrations/949 The Titanic II: S.CP.2-6 http://www.illustrativemathematics.org/illustrations/950 Return to Fred's Fun Factory (with 50 cents): S.CP.1,2, and 9 http://www.illustrativemathematics.org/illustrations/1198 Law of Sines and Law of Cosines: http://illuminations.nctm.org/LessonDetail.aspx?ID=L703

DIFFERENTIATION		
UDL/FRONT LOADING	**ACCELERATION**	**INTERVENTION**
Prerequisites: • Review and have students provide examples of proportion and ratios. They can construct a 3-D solid and copy within specific proportions. • Have students should review similar triangles. • Vocabulary should be reviewed • Engage students in a discussion about planes versus space (2-D versus 3-D) as well as area formulas and how to use them to find the volume formulas. • Use T-chart or other graphic organizer to compare Independent Events and Dependent Events. Use Frayer model to provide the definition of probability.	• Design an activity where students would collect data and then use probability model to interpret the data. **For example**, students can collect data to answer the following real-life question: There is little doubt that caffeine stimulates bodily activity, but how much does it take to produce a significant effect? This is a question that involves measuring the effect of two or more treatments and deciding if the different interventions have differing effects. To obtain a partial answer to the question on caffeine, it was decided to compare a treatment consisting of 200 mg of caffeine with a control of no caffeine in an experiment involving a finger tapping exercise. Twenty male students were randomly assigned to one of two treatment groups of 10 students each, one group receiving 200-milligram of caffeine and the other group no caffeine. Two hours later the students were given a finger tapping exercise. The response is the number of taps per minute, as shown in the table.	• Hands-on 3 D solids that allow student to have the visual to understand different parts and vocabulary of volumes. • Also the hands on will allow volume comparison. • Interactive online websites describing the changes in ratio with changing dimensions. • Scaffolding • Vocabulary wall • To increase active participation, students should be expected to work collaboratively to help language learners with lowering anxiety, promote authentic conversation, opportunities for asking questions, and support peers and teachers. Alex, Mel, and Chelsea play a game that has 6 rounds. In each round there is a single winner, and the outcomes of the rounds are independent. For each round the probability that Alex wins is 12, and Mel is twice as likely to win as Chelsea. What is the probability that Alex wins three rounds, Mel wins two rounds, and Chelsea wins one round? http://www.illustrativemathematics.org/illustrations/1035

<table>
<tr><th colspan="3">DIFFERENTIATION📖</th></tr>
<tr><th>UDL/FRONT LOADING</th><th>ACCELERATION</th><th>INTERVENTION</th></tr>
<tr><td></td><td>
<table>
<tr><th colspan="3">Finger taps per minute in a caffeine experiment</th></tr>
<tr><th></th><th>0 mg caffeine</th><th>200 mg caffeine</th></tr>
<tr><td></td><td>242</td><td>246</td></tr>
<tr><td></td><td>245</td><td>248</td></tr>
<tr><td></td><td>244</td><td>250</td></tr>
<tr><td></td><td>248</td><td>252</td></tr>
<tr><td></td><td>247</td><td>248</td></tr>
<tr><td></td><td>248</td><td>250</td></tr>
<tr><td></td><td>242</td><td>246</td></tr>
<tr><td></td><td>244</td><td>248</td></tr>
<tr><td></td><td>246</td><td>245</td></tr>
<tr><td></td><td>242</td><td>250</td></tr>
<tr><td>Mean</td><td>244.8</td><td>248.3</td></tr>
</table>
Source: Draper and Smith, Applied Regression Analysis, John Wiley and Sons, 1981

Students use trigonometric functions to find dimensions or distances of objects in real life. For instance: Around 1:30 p.m., people heard that the space shuttle will fly around Los Angeles area. People were outside waiting. Finally, the space shuttle is observed. At one point, it appeared as if the shuttle was really low. The observer’s distance is about 100 feet away from it (diagonal distance) with an angle of elevation of 30^o. How high is the shuttle from the ground?
Use the following activity which requires students to identify whether or not a game is fair: http://insidemathematics.org/problems-of-the-month/pom-gameshow.pdf

</td><td></td></tr>
</table>

[1] **Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.**

[2] **Supporting/Additional Clusters – designed to support and strengthen areas of major emphasis/expose students to other subjects.**

References:

1. National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards (Mathematics)*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.

2. McCallum, W., Zimba, J., Daro, P. (2011, December 26 Draft). *Progressions for the Common Core State Standards in Mathematics*. Cathy Kessel (Ed.). Retrieved from http://ime.math.arizona.edu/progressions/#committee.
3. Engage NY. (2012). New York Common Core Mathematics Curriculum. Retrieved from http://www.engageny.org/resource/high-school-geometry.
4. Mathematics Assessment Resource Service, University of Nottingham. (2007 - 2012). Mathematics Assessment Project. Retrieved from http://map.mathshell.org/materials/index.php.
5. Smarter Balanced Assessment Consortium. (2012). Smarter Balanced Assessments. Retrieved from http://www.smarterbalanced.org/.
6. Partnership for Assessment of Readiness for College and Career. (2012). PARCC Assessments. Retrieved from http://www.parcconline.org/parcc-assessment.
7. California Department of Education. (2013). Draft Mathematics Framework Chapters. Retrieved from http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp.
8. National Council of Teachers of Mathematics (NCTM) Illuminations. (2013). Retrieved from http://illuminations.nctm.org/Weblinks.aspx.
9. The University of Arizona. (2011-12). Progressions Documents for the Common Core Math Standards. Retrieved from http://ime.math.arizona.edu/progressions.